OPTIMIZING A THERMOANEMOMETRIC EXPERIMENT AT LOW AND HIGH TURBULENCE INTENSITIES

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UDC 536.6.08

Precise A-optimum and partly optimum plans are constructed for a thermoanemometric experiment to determine components of the Reynolds stress tensor.

Turbulence characteristics are determined with a hot-wire anemometer by conducting a series of measurements with the wire located at different positions relative to the mean velocity vector and overheated by different amounts relative to the environment [1-4]. The dependence of the experimental results on a large number of factors which can be assigned with a finite accuracy affects the measurement error. The latter can thus be appreciable [1, 2, 4] in certain cases (shear flows with high-intensity pulsations, separation zones, etc.). In connection with this, it becomes important to look for the conditions that will ensure the assigned degree of accuracy. Below we examine problems of optimizing a thermoanemometric experiment in flows with a relatively low (Tu $\leqslant$ 0.2) and fairly high (Tu > 0.2) intensity of turbulence.

With a relatively low level of pulsations, turbulence characteristics can be found on the basis of a theory whereby the components of the actual velocity vector are related as follows to the effective velocity (it is assumed that the wire is not sensitive to tangential flows) [1, 2]:

$$Q_i = (U + u)\sin\alpha_i + v\cos\alpha_i. \tag{1}$$

Equation (1) is a linear regression with a controlled variable α_i and unknown parameters U + u, v. Determination of the velocity vector components reduces to measuring values of Q_i at different positions of the transducer (i = 1, ..., N) and subsequent solution of a system of N regression equations (1). The approximate solution of such a system has the form [5]

$$T_k = \sum_{j=1}^{m} \sum_{i=1}^{N} (M^{-1})_{kj}\, \sigma_i^{-2} f_{ji} Q_i \quad (k = 1, \ldots, m). \tag{2}$$

In the case of independent measurement of all Q_i, the dispersions of the sought parameters which determine their errors are the diagonal elements of the dispersion matrix $D = M^{-1}$. They are equal to the following ($N = m = 2$, $\sigma_1 = \sigma_2 = \sigma$):

$$D_{11} = \sigma^2 \frac{\cos^2\alpha_1 + \cos^2\alpha_2}{\sin^2(\alpha_1 - \alpha_2)}, \quad D_{22} = \sigma^2 \frac{\sin^2\alpha_1 + \sin^2\alpha_2}{\sin^2(\alpha_1 - \alpha_2)}. \tag{3}$$

Equations (3) allow us to find the conditions for minimum error in measuring the individual components of the velocity vector. To determine the entire group of sought parameters with the required degree of accuracy, it is best to optimize in accordance with the A criterion, i.e., it is best to minimize Sp D [5]. In this case, the angles α_i should satisfy the condition $|\alpha_1 - \alpha_2| = \frac{\pi}{2}$.*

Mean-square values of the effective velocity, rather than actual values of the latter, are generally determined in thermoanemometric measurements. In this case, the method of "three rotations" allows us to find the second moments of the velocity field. The equa-

*The A-optimum plan corresponds to the placement of a cross-shaped transducer in the uv plane.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 40, No. 6, pp. 975-979, June, 1981. Original article submitted April 4, 1980.

TABLE 1. Spectra of Optimum Plans

α_i, D_{kk}	Criteria				
	single-wire transducer			two-wire transducer	
	min D_{11}	min D_{22}	min D_{33}	min Sp D	min D_{33}
α_1/π	0,5	0	0,25	0,5	0,25
α_2/π	0,5	0	—0,25	0	—0,25
α_3/π	0,5	0	—	—	—
$\sqrt{D_{11}}/\sigma$	0,58	∞	1	1	1,22
$\sqrt{D_{22}}/\sigma$	∞	0,58	1	1	1,22
$\sqrt{D_{33}}/\sigma$	∞	∞	0,71	1	0,71

TABLE 2. A-Optimum and Partial-Optimum Plans

i	Criteria											
	min Sp D			min D_{11}			min D_{33}			min Sp D ($m=6$)		
	$\frac{\Theta_i}{\pi}$	$\frac{\varphi_i}{\pi}$	$\frac{\sqrt{D_{ii}}}{\sigma}$	$\frac{\Theta_i}{\pi}$	$\frac{\varphi_i}{\pi}$	$\frac{\sqrt{D_{ii}}}{\sigma}$	$\frac{\Theta_i}{\pi}$	$\frac{\varphi_i}{\pi}$	$\frac{\sqrt{D_{ii}}}{\sigma}$	$\frac{\Theta_i}{\pi}$	$\frac{\varphi_i}{\pi}$	$\frac{\sqrt{D_{ii}}}{\sigma}$
1	0,5	0	1,0	0,5	0,52	0,82	0,5	0,28	4,58	0,41	0,05	0,88
2	0,5	0,38	0,86	0,5	0,49	0,82	0,5	0,69	4,07	0,44	1,35	0,96
3	0,5	0,69	0,91	0,5	0	26	0,5	0,75	0,65	0,43	1,68	0,87
4	0,089	0	0,81	0	—	0,82	0,11	1,0	1,25	0,11	1,58	0,94
5	0,19	1,0	1,0	0,25	0	1,28	0,1	0	1,31	0,31	0,98	0,94
6	—	—	—	—	—	—	—	—	—	0,24	0,48	0,83

TABLE 3. Nonoptimum Plans

i	$m=5$					
	I			II		
	$\frac{\Theta_i}{\pi}$	$\frac{\varphi_i}{\pi}$	$\frac{\sqrt{D_{ii}}}{\sigma}$	$\frac{\Theta_i}{\pi}$	$\frac{\varphi_i}{\pi}$	$\frac{\sqrt{D_{ii}}}{\sigma}$
1	0,5	0,25	2,26	0,5	0,28	4,76
2	0,5	0,75	0,93	0,5	0,69	3,77
3	0,5	0,5	0,73	0,5	0,41	1,07
4	0,25	0	1,74	0,22	0,50	1,55
5	0,25	1	0,73	0,19	1	1,39

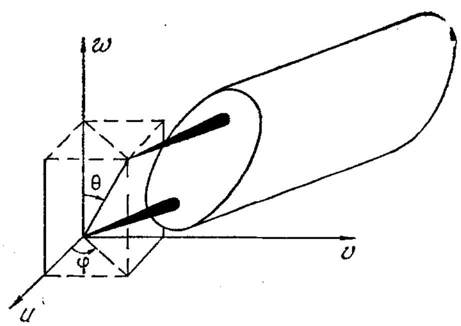

Fig. 1. Coordinate system.

tion connecting the mean square value of Q_i with the Reynolds stress components has the form

$$\langle (Q_i - \langle Q_i \rangle)^2 \rangle = \langle u^2 \rangle \sin^2\alpha_i + \langle v^2 \rangle \cos^2\alpha_i + \langle uv \rangle \sin 2\alpha_i. \quad (4)$$

The diagonal elements of the dispersion matrix (N = m = 3) are expressed through the following relations:

$$D_{11} = \sigma^2 [\cos^2\alpha_1 \cos^2\alpha_2 \sin^2(\alpha_1 - \alpha_2) + \cos^2\alpha_1 \cos^2\alpha_3 \sin^2(\alpha_1 - \alpha_3) + \cos^2\alpha_2 \cos^2\alpha_3 \sin^2(\alpha_2 - \alpha_3)]/[\sin^2(\alpha_1 - \alpha_2) \sin^2(\alpha_1 - \alpha_3) \sin^2(\alpha_2 - \alpha_3)], \quad (5)$$

$$D_{22}=\sigma^2[\sin^2\alpha_1\sin^2\alpha_2\sin^2(\alpha_1-\alpha_2)+\sin^2\alpha_1\sin^2\alpha_3\sin^2(\alpha_1-\alpha_3)+ \quad (6)$$

$$+\sin^2\alpha_2\sin^2\alpha_3\sin^2(\alpha_2-\alpha_3)]/[\sin^2(\alpha_1-\alpha_2)\sin^2(\alpha_1-\alpha_3)\sin^2(\alpha_2-\alpha_3)],$$

$$D_{33}=\sigma^2[\sin^2(\alpha_2-\alpha_3)\sin^2(\alpha_2+\alpha_3)+\sin^2(\alpha_1-\alpha_3)\sin^2(\alpha_1+\alpha_3)+ \quad (7)$$

$$+\sin^2(\alpha_1-\alpha_2)\sin^2(\alpha_1+\alpha_2)]/[\sin^2(\alpha_1-\alpha_3)\sin^2(\alpha_1-\alpha_3)\sin^2(\alpha_2-\alpha_3)].$$

For regression (4), the spectrum of the A-optimum plan consists of the points

$$\alpha_3=\alpha_1-\frac{\pi}{3},\quad \alpha_2=\alpha_1+\frac{\pi}{3}$$

(α_1 is an arbitrary point). Here, the dispersions $D_{11}=D_{22}$ and D_{33} are, respectively, equal to σ^2 and $\frac{2}{3}\sigma^2$, , while $\mathrm{Sp}\,D=\frac{8}{3}\sigma^2$.

The results shown pertain to optimization of the thermoanemometric experiment as a whole. In a practical sense, it is also of interest to improve the accuracy of the determination of the individual turbulence characteristics $\langle u^2 \rangle$, $\langle v^2 \rangle$, etc. Table 1 shows spectra of the corresponding optimum plans and the measurement errors of single-wire ("three rotations" method) and two-wire (with angles α_1 and α_2 relative to the mean velocity vector) transducers.

With high-intensity pulsations, the systematic error associated with linearization of the heat balance equation of the wire increases markedly. This limits the range of values of Tu (Tu $\leqslant$ 0.2) in which Eqs. (1) and (4) can be used. Another approach based on measurement of the mean square of the effective velocity [6, 7] is used to determine turbulence characteristics within the interval Tu > 0.2. In a spherical coordinate system (Fig. 1), the corresponding system of regression equations has the form*

$$Q_i=f_{1i}\langle(U+u)^2\rangle+f_{2i}\langle v^2\rangle+f_{3i}\langle uv\rangle+f_{4i}\langle w^2\rangle+f_{5i}\langle uw\rangle+f_{6i}\langle vw\rangle, \quad (8)$$

where

$$f_{1i}=1+(k^2-1)\sin^2\Theta_i\cos^2\varphi_i;\quad f_{2i}=1+(k^2-1)\sin^2\Theta_i\sin^2\varphi_i;$$

$$f_{3i}=(k^2-1)\sin^2\Theta_i\sin 2\varphi_i;\quad f_{4i}=1+(k^2-1)\cos^2\Theta_i;$$

$$f_{5i}=(k^2-1)\sin 2\Theta_i\cos\varphi_i;\quad f_{6i}=(k^2-1)\sin 2\Theta_i\sin\varphi_i.$$

In this case, optimizing the experiment reduces to finding the minimum of the vector functional of the dispersion matrix, the specific form of which depends on the formulation of the problem being studied. Table 2 shows spectra of optimum plans (m = 5, $\langle vw \rangle = 0$ and m = 6, $\langle vw \rangle \neq 0$; N = m) obtained as a result of approximate calculations. These calculations showed that the errors associated with measurement of the angles can be neglected, since they are very small and only slightly affect the value of D_{kk}. For the sake of comparison, Table 3 shows data on measurement errors at random positions of the transducer. It is apparent that optimizing the experiment significantly improves the accuracy of the measurements.

NOTATION

Tu, intensity of turbulence; Q_i, measured value in the i-th test (effective velocity or mean value of its square); U, mean velocity; u, v, w, components of the eddy velocity; α_i, angle between the direction of a wire located in the plane uv and the mean velocity vector; N, number of measurements; m, number of unknown parameters; M, Fisher data matrix; D, dispersion matrix; T_k, linear estimate of the k-th parameter; f_{ji}, j-th component of the vector function of the factors α_i or Θ_i, φ_i; σ_i, error of measurement of Q_i; $\langle\ \rangle$, average over time; $D_{11}, \ldots, D_{66}$, respectively, the dispersions of the quantities $\langle (U+u)^2 \rangle$, $\langle v^2 \rangle$, $\langle uv \rangle$, $\langle w^2 \rangle$, $\langle uw \rangle$, $\langle vw \rangle$.

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CLOSED EQUATION FOR THE STRUCTURE FUNCTION OF AN ISOTROPIC TURBULENT VELOCITY FIELD

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UDC 532.517.4

A closed equation is derived for the structure function of an isotropic turbulent velocity field in an incompressible fluid. The equation for the characteristic function [1] is used as the initial equation.

A closed equation was obtained in [1] for the characteristic function φ of the probability distribution of the differences in velocities and temperatures at two points in an isotropic turbulent flow of an incompressible fluid. Here, we use this equation as the initial equation to derive an equation for the structure function, which is defined as follows:

$$D(r,\ t) = \langle \Delta V_l(r,\ t)^2 \rangle . \tag{1}$$

The time-dependent argument of the structure functions will not be indicated in subsequent discussions.

Using the equation for φ, we can also obtain an equation for the structure function of the temperature field H(r, t). This equation will be derived in the present article. In making the transition from the equation for φ to the equations for D and H, there again arises the problem of closure. To obtain closed equations for D and H, we need to make certain assumptions regarding the form of the characteristic function φ. We will choose for the form of φ the product of the Gaussian characteristic function and an expression accounting for the deviation of the combined probability distributions of the velocity and temperature differences from the normal distribution. This expression will contain only those moments of the probability distribution which have an important physical significance: 1) the double-point third-order structural tensor $D_{ijl}(r)$ describing the transfer of energy between different-scale pulsations of the turbulent velocity field,

$$D_{ijl}(\mathbf{r}) = \langle \Delta V_i(\mathbf{r}) \Delta V_j(\mathbf{r}) \Delta V_l(\mathbf{r}) \rangle ; \tag{2}$$

2) a mixed third-order moment defining turbulent mixing of the temperature field

$$D_{jTT}(\mathbf{r}) = \langle \Delta V_j(\mathbf{r}) \Delta T^2(\mathbf{r}) \rangle . \tag{3}$$

Thus, the assumption made with regard to the form of φ consists of the following:

$$\varphi_{\mathbf{r},t}(\theta,\ \eta) = \exp \Gamma(\theta,\ \eta) [1 - iT(\theta,\ \eta)], \tag{4}$$

where

$$\Gamma(\theta,\ \eta) = -\frac{1}{2} D_{ij}(\mathbf{r}) \theta_i \theta_j - \frac{1}{2} H(\mathbf{r}) \eta^2; \tag{5}$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 40, No. 6, pp. 980-992, June, 1981. Original article submitted May 17, 1979.

 0022-0841/81/4006-0584$07.50